The derivative as a rate of change

MATH 201 Calculus I, week 6
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1 What you should be able to do after this week

- Say what a derivative measures, in a sentence.
- Find the derivative of a polynomial.
- Read the slope of a curve from its graph.

2 The idea

The average rate of change of a function f between x and $x+h$ is the slope of the line through the two points. The derivative is what that slope approaches as h shrinks to zero:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}. \tag{1}$$

2.1 A worked example

Take $f(x) = x^2$. Then

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = 2x + h, \tag{2}$$

and as h goes to zero this becomes $2x$. So $f'(x) = 2x$: at $x = 1$ the curve rises 2 units for each unit to the right.

3 Rules to remember

Function	Derivative
c (a constant)	0
x^n	$n x^{n-1}$
$c\,f(x)$	$c\,f'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$

4 Practice

1. Find $f'(x)$ for $f(x) = 3x^2 - 5x + 1$.
2. Where is the slope of $y = x^3 - 3x$ equal to zero?
3. A ball's height in metres after t seconds is $h(t) = 20t - 5t^2$. How fast is it rising after one second?

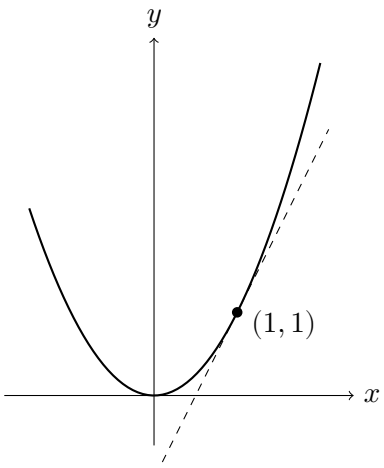


Figure 1: The parabola $y = x^2$ and its tangent at $(1, 1)$, which has slope 2.

5 Where to read more

Textbook Chapter 3, sections 3.1 to 3.3.

Online The OpenStax Calculus Volume 1 book is free to read.

Office hours Tuesdays 14:00 to 16:00, room 214.