APEX ENGINEERING INSTITUTE
Department of Mechanical & Civil Engineering

ENGR 230: Engineering Statics & Dynamics

Problem Set 4: Rigid Body Equilibrium & Particle Kinematics

Date Issued: Oct 7, 2026
Due Date: Oct 14, 2026
Instructor: Prof. Marcus Vance
Total Points: 100 Points

Student Name: ______________ **Student ID:** ______________ **Section / Recitation:** ______________

Submission Guidelines & Policy: Show all free-body diagrams (FBDs), coordinate axis definitions, vector components, and intermediate algebraic steps clearly. State all physical assumptions (e.g., rigid members, frictionless pins, uniform gravitational field with $g = 9.81\,\mathrm{m/s^2}$). Express final numerical answers with appropriate physical units and 3 significant figures.

Section 1: Statics & Rigid Body Equilibrium

Problem 1: Planar Bridge Truss Analysis & Zero-Force Members [25 Points]

A pin-connected structural bridge truss is subjected to vertical concentrated loads from deck traffic at joints B and C, as depicted in Figure 1. The structure is supported by a smooth pin at joint A and a roller support on a horizontal surface at joint E. All structural members are composed of high-strength structural steel channels ($E = 200\,\text{GPa}$).

Given Data & Parameters

Load $P_1 = 18.0\,\text{kN}$ applied downwards at joint B.
Load $P_2 = 30.0\,\text{kN}$ applied downwards at joint C.
Horizontal bay lengths: $L_{AB} = L_{BC} = L_{CD} = L_{DE} = 3.0\,\text{m}$.
Vertical height of top chord joints F and G: $H = 4.0\,\text{m}$.

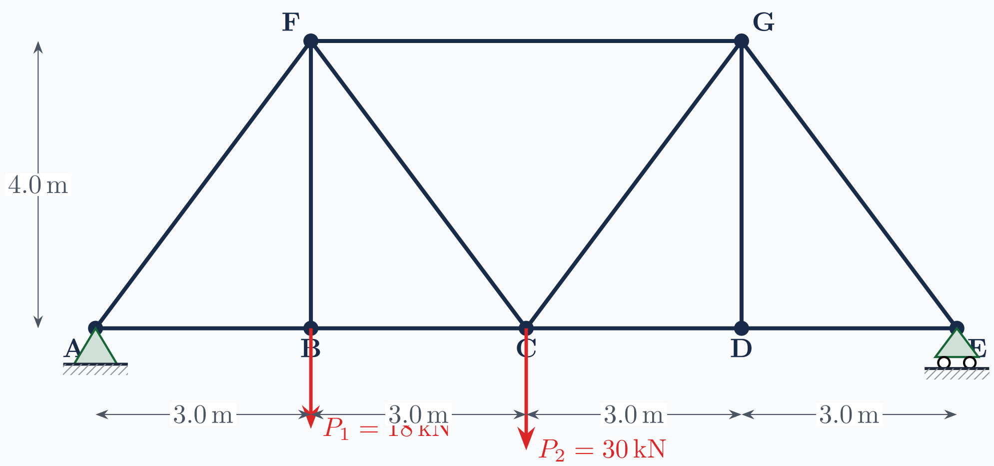

Figure 1: Pin-connected planar bridge truss loading configuration and geometry.

a[1] Determine the external support reaction forces at pin support A (A_x, A_y) and roller support E (E_y).

b[2] Identify any zero-force members present in the truss under the specified load condition by inspection, providing a brief mechanical justification for each.

c[3] Using the **Method of Joints**, calculate the internal axial forces in members AF, BF, FC, and FG. State whether each member is in tension (T) or compression (C).

Solution Workspace: Truss Support Reactions & Member Force Calculations

(a) Global Equilibrium Equations:

$$\sum F_x = 0 \implies A_x = 0$$
$$\sum M_A = 0 \implies E_y(12.0) - P_1(3.0) - P_2(6.0) = 0$$
$$\implies E_y(12.0) - (18.0)(3.0) - (30.0)(6.0) = 0 \implies E_y = 19.5\,\text{kN}\uparrow$$
$$\sum F_y = 0 \implies A_y + E_y - P_1 - P_2 = 0 \implies A_y = 48.0 - 19.5 = 28.5\,\text{kN}\uparrow$$

Problem 2: Dry Friction on Inclined Plane with Cable Tension [25 Points]

A heavy modular equipment crate of mass $m = 65.0\,\text{kg}$ rests on a rough inclined ramp inclined at an angle $\theta = 28°$ above the horizontal. A winch cable attached to the front of the crate exerts a tension force T directed at an angle $\phi = 12°$ relative to the inclined surface, as illustrated in Figure 2. The static coefficient of friction between the crate and ramp is $\mu_s = 0.38$, and the kinetic coefficient of friction is $\mu_k = 0.28$.

Given Data & Parameters

Mass of crate $m = 65.0\,\text{kg}$; Gravitational acceleration $g = 9.81\,\text{m/s}^2$.
Incline angle $\theta = 28°$; Cable angle relative to ramp surface $\phi = 12°$.
Coefficients of friction: Static $\mu_s = 0.38$, Kinetic $\mu_k = 0.28$.

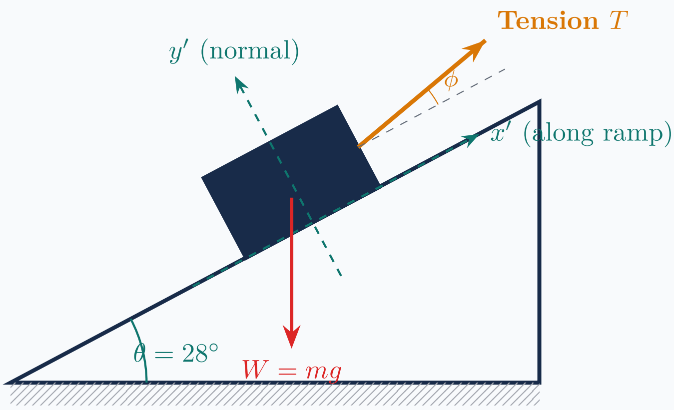

Figure 2: Equipment crate on an inclined ramp subjected to cable tension force T.

a[1] Draw a complete, fully labeled **Free-Body Diagram (FBD)** of the crate, defining an inclined coordinate system (x', y') aligned parallel and perpendicular to the ramp surface.

b[2] Determine the minimum tension force T_{min} required to **prevent the crate from slipping down** the ramp.

c[3] Determine the tension force T_{imp} required to **impend motion up** the ramp.

d[4] If the cable suddenly snaps ($T = 0$), determine whether the crate remains at rest or slides down the ramp. If it slides, calculate its acceleration a_x down the incline.

Solution Workspace: Free-Body Diagram & Impending Motion Equations

(a) Free-Body Diagram Equations along inclined axes (x', y'):

$$\sum F_{x'} = 0 \implies T\cos\phi + f_s - mg\sin\theta = 0$$
$$\sum F_{y'} = 0 \implies N + T\sin\phi - mg\cos\theta = 0 \implies N = mg\cos\theta - T\sin\phi$$

Section 2: Dynamics of Particles & Rigid Bodies

Problem 3: Particle Projectile Kinematics with Drag [25 Points]

An autonomous research drone developed at Meridian Dynamics launches an environmental sensor package from the edge of a cliff at height $h = 45.0\,\mathrm{m}$ above a valley floor, as illustrated in Figure 3. The initial launch speed is $v_0 = 32.0\,\mathrm{m/s}$ at an angle $\alpha = 35.0°$ above the horizontal.

Given Data & Parameters

Initial cliff elevation $h = 45.0\,\mathrm{m}$; Initial speed $v_0 = 32.0\,\mathrm{m/s}$; Angle $\alpha = 35.0°$.
Gravitational acceleration $g = 9.81\,\mathrm{m/s^2}$ directed vertically downwards.

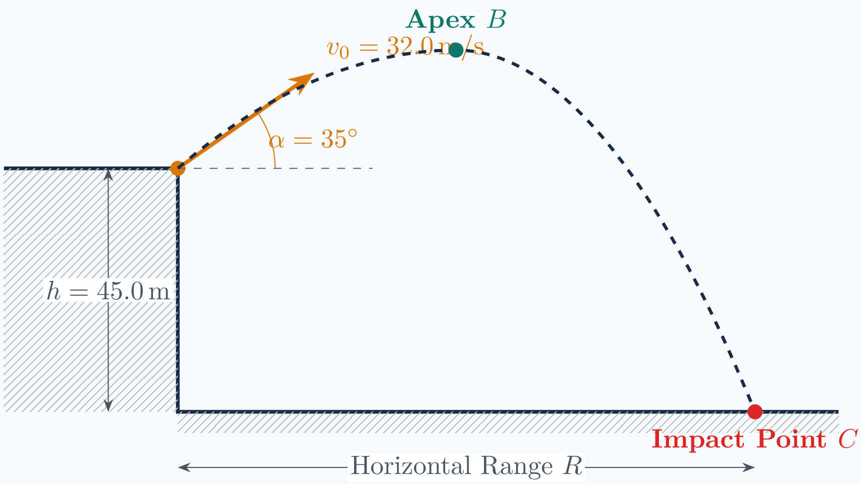

Figure 3: Autonomous sensor package projectile trajectory launched from cliff apex.

a[1] Neglecting air resistance, calculate the maximum peak altitude y_{max} reached by the payload measured from the valley floor.

b[2] Calculate the total time of flight t_{impact} until the payload impacts the valley floor at point C.

c[3] Determine the horizontal range R from the base of the cliff to the impact point C, and the magnitude and direction of the velocity vector $\vec{v}_C$ immediately prior to impact.

Solution Workspace: Kinematic Component Equations & Trajectory Integration

Initial Velocity Components:

$$v_{0x} = v_0 \cos\alpha = 32.0\cos(35°) = 26.21\,\mathrm{m/s}$$
$$v_{0y} = v_0 \sin\alpha = 32.0\sin(35°) = 18.35\,\mathrm{m/s}$$

Problem 4: Planar Kinetics of Rigid Bodies & Stepped Pulley [25 Points]

A dual-concentric stepped pulley assembly of mass $m_P = 16.0\,\mathrm{kg}$ and mass radius of gyration $k_O = 0.22\,\mathrm{m}$ about its central fixed axis O is mounted on frictionless bearings. Block A ($m_A = 12.0\,\mathrm{kg}$) is suspended from an inextensible light cable wrapped around the outer drum of radius $R = 0.35\,\mathrm{m}$. A constant tension force $P = 85.0\,\mathrm{N}$ is applied to a second cable wrapped around the inner hub of radius $r = 0.18\,\mathrm{m}$, as shown in Figure 4.

Given Data & Parameters

Pulley mass $m_P = 16.0\,\mathrm{kg}$; Radius of gyration $k_O = 0.22\,\mathrm{m}$; Outer radius $R = 0.35\,\mathrm{m}$; Hub radius $r = 0.18\,\mathrm{m}$.
Suspended mass $m_A = 12.0\,\mathrm{kg}$; Pull force $P = 85.0\,\mathrm{N}$; Gravitational acceleration $g = 9.81\,\mathrm{m/s^2}$.

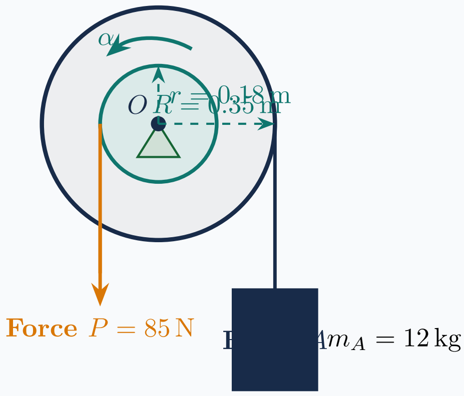

Figure 4: Concentric stepped pulley dynamics with applied cable force and suspended load.

a[1] Compute the mass moment of inertia I_O of the stepped pulley system about its central axis O.

b[2] Draw individual Free-Body Diagrams (FBDs) for both the pulley system and Block A.

c[3] Formulate Newton-Euler equations of motion ($\sum M_O = I_O\alpha$ and $\sum F_y = m_A a_A$) to calculate the angular acceleration α of the pulley and the linear acceleration a_A of Block A.

Solution Workspace: Moment of Inertia & Equations of Motion Integration

(a) Mass Moment of Inertia Calculation:

$$I_O = m_P\, k_O^2 = (16.0\,\mathrm{kg})\,(0.22\,\mathrm{m})^2 = 0.7744\,\mathrm{kg\,m^2}$$

Section 3: Reference & Grading Breakdown

Statics Formula Quick Reference

2D Equilibrium Conditions:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_O = 0$$

Dry Friction Laws (Coulomb):

$$F_{s,\max} = \mu_s N, \quad F_k = \mu_k N$$

Truss Zero-Force Member Rules:

- Two non-collinear members at unloaded joint $\implies$ both zero.
- Three members at unloaded joint, two collinear $\implies$ 3rd is zero.

Dynamics Formula Quick Reference

Kinematics (Constant Acceleration):

$$v(t) = v_0 + at, \quad s(t) = s_0 + v_0 t + \frac{1}{2}at^2$$

Rigid Body Planar Kinetics:

$$\sum \vec{F} = m\vec{a}_G, \quad \sum M_O = I_O \alpha$$

Radius of Gyration:

$$I_O = mk_O^2, \quad a_t = r\alpha$$

Evaluation & Grading Rubric

Problem No.	Max Points	Core Competencies Evaluated	Score
Problem 1	25	Global reactions, zero-force member identification, method of joints	/ 25
Problem 2	25	FBD precision, inclined axes resolution, static/kinetic friction threshold	/ 25
Problem 3	25	2D particle kinematics, elevated launch trajectory, vector velocity	/ 25
Problem 4	25	Rotational inertia, moment of inertia, combined cable-pulley kinetics	/ 25
TOTAL	**100**	**Overall Engineering Statics & Dynamics Performance**	**/ 100**