

Precision Gauge Theory Simulations and Non-Perturbative Parton Structure at the Meridian High-Energy Facility

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Abstract

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We present high-precision non-perturbative calculations of hadronic matrix elements and parton distribution functions derived from lattice gauge theory simulations at physical quark masses. Using gauge field configurations generated on the Vertex Parallel Supercomputing Cluster with a tree-level improved Symanzik gauge action and $N_f = 2 + 1 + 1$ flavors of highly improved staggered quarks (HISQ), we compute the pion and nucleon vector and axial-vector form factors across momentum transfers up to $Q^2 = 3.0 \text{ GeV}^2$. Continuum extrapolations are performed across three lattice spacings ($a \approx 0.06, 0.09, 0.12 \text{ fm}$) with physical spatial volumes satisfying $m_\pi L > 4.0$. Hadronic matrix elements are renormalized non-perturbatively via the regularized RI'-MOM scheme and converted to the $\overline{\text{MS}}$ scheme at 2 GeV . Our results for the nucleon axial charge $g_A = 1.268(14)$ and the pion decay constant $f_\pi = 130.4(9) \text{ MeV}$ demonstrate sub-percent control over systematic uncertainties, providing key theoretical inputs for ongoing and future scattering experiments at the Meridian Collider Facility.

1 Introduction

Understanding the non-perturbative structure of hadrons from first principles in Quantum Chromodynamics (QCD) remains one of the primary goals of modern subatomic physics. While high-energy hard scattering processes are well described by perturbative QCD expansions in powers of the strong coupling constant $\alpha_s(Q^2)$, low-energy observables and bound-state properties necessitate non-perturbative techniques. Lattice QCD provides a mathematically rigorous, non-perturbative regularization of the gauge theory formulated on a four-dimensional Euclidean spacetime grid.

In recent years, advances in algorithmic design—such as adaptive algebraic multigrid Dirac solvers

and Fourier-accelerated Hybrid Monte Carlo (HMC) algorithms—combined with high-performance computing architectures have enabled lattice simulations to operate directly at physical light-quark masses. This eliminates the reliance on chiral extrapolations from unphysically heavy pion masses, which historically served as a major source of systematic error.

This report summarizes recent progress achieved by the Nexa-Vertex-Synapse Lattice Collaboration in evaluating hadron matrix elements relevant for lepton-hadron scattering experiments conducted at the Meridian High-Energy Collider. Special emphasis is placed on controlling continuum, finite-volume, and excited-state contamination.

2 Theoretical Framework

The Euclidean action for lattice gauge theory consists of the pure gauge action $S_G[U]$ and the fermion action $S_F[\psi, \bar{\psi}, U]$:

$$S[U, \psi, \bar{\psi}] = S_G[U] + S_F[\psi, \bar{\psi}, U]. \quad (1)$$

For the gauge sector, we employ the tree-level improved Symanzik action defined on the hypercubic lattice $\Lambda = \{x = (x_1, x_2, x_3, x_4) \mid x_\mu/a \in \mathbb{Z}\}$:

$$S_G[U] = \frac{\beta}{3} \sum_{x, \mu < \nu} \left(c_0 \text{Re Tr} [1 - P_{\mu\nu}(x)] + c_1 \text{Re Tr} [1 - R_{\mu\nu}(x)] \right), \quad (2)$$

where $P_{\mu\nu}(x)$ denotes the standard 1×1 plaquette loop, $R_{\mu\nu}(x)$ represents the rectangular 1×2 loop, $\beta = 6/g^2$ is the lattice gauge coupling, and the normalization condition requires $c_0 + 8c_1 = 1$ with $c_1 = -1/12$.

The fermion action uses the $N_f = 2 + 1 + 1$ Highly Improved Staggered Quark (HISQ) formulation for sea quarks and the Wilson-clover action for valence

Table 1: Parameters of the $N_f = 2+1+1$ lattice gauge ensembles generated by the Nexa-Vertex collaboration. Listed are the lattice coupling β , volume $(L/a)^3 \times (T/a)$, lattice spacing a , pion mass m_π , and total HMC trajectories.

Ensemble	β	Volume	a (fm)	m_π (MeV)	Traj.
NV-A06	6.30	$64^3 \times 128$	0.058	134.2	8,400
NV-A09a	6.00	$48^3 \times 96$	0.088	135.1	10,200
NV-A09b	6.00	$32^3 \times 64$	0.088	220.4	12,000
NV-A12a	5.80	$32^3 \times 64$	0.121	136.0	9,600
NV-A12b	5.80	$24^3 \times 48$	0.121	310.8	14,500

u, d, and s quarks. The Wilson-clover Dirac operator D_{clover} is given by:

$$D_{\text{clover}}(x, y) = \left(m_0 + \frac{4}{a}\right) \delta_{x, y} - \frac{1}{2a} \sum_{\mu=1}^4 \left[(1 - \gamma_\mu) U_\mu(x) \delta_{x+a\hat{\mu}, y} + (1 + \gamma_\mu) U_\mu^\dagger(y) \delta_{x-a\hat{\mu}, y} \right] + c_{\text{SW}} \frac{a}{4} \sum_{\mu, \nu} \sigma_{\mu\nu} F_{\mu\nu}(x) \delta_{x, y}, \quad (3)$$

where c_{SW} is the Sheikholeslami-Wohlert coefficient tuned non-perturbatively to eliminate $\mathcal{O}(a)$ discretization errors, and $F_{\mu\nu}(x)$ is the lattice field strength tensor constructed from clover-shaped four-plaquette loops.

3 Computational Methodology

Gauge field ensembles were generated using the Hasenbusch-preconditioned Hybrid Monte Carlo algorithm integrated with an adaptive leapfrog integrator. The numerical inversions of the Dirac operator were accelerated using an algebraic multigrid preconditioner implemented on the Apex-64 GPU cluster hosted at the Vertex Research Laboratory.

Table 1 outlines the key parameters of the five gauge configurations used in this analysis. Spatial volumes $L^3 \times T$ were chosen such that $m_\pi L \geq 4.0$ across all ensembles to suppress finite-volume effects to below 0.2%.

Correlator measurements were carried out using momentum-smeared point and gaussian sources. Ground-state isolation was achieved by performing multi-state exponential fits to three-point correlation functions over multiple source-sink separations $\tau \in [0.8, 1.6]$ fm.

Table 2: Systematic error budget for the nucleon axial charge g_A and pion decay constant f_π evaluated in this work.

Source of Uncertainty	$\Delta g_A / g_A$ (%)	$\Delta f_\pi / f_\pi$ (%)
Statistical	0.72	0.45
Continuum Limit ($a \rightarrow 0$)	0.55	0.38
Chiral Interpolation	0.31	0.20
Finite Volume ($m_\pi L$)	0.18	0.12
Renormalization (Z_A, Z_V)	0.40	0.30
Excited-State Effects	0.50	0.25
Total Uncertainty	1.10	0.69

4 Results and Error Budget

Hadronic matrix elements $\langle H(p') | \mathcal{O}_\mu | H(p) \rangle$ were extracted from ratios of three-point to two-point correlation functions:

$$R_{\mathcal{O}}(t, \tau) = \frac{C_{\mathcal{O}}^{(3)}(t, \tau; \vec{p}', \vec{p})}{C^{(2)}(\tau; \vec{p})} [\mathcal{K}(t, \tau; \vec{p}', \vec{p})]^{1/2}, \quad (4)$$

where the kinematic ratio factor $\mathcal{K}$ is defined as:

$$\mathcal{K} = \frac{C^{(2)}(t; \vec{p}') C^{(2)}(\tau; \vec{p}') C^{(2)}(\tau - t; \vec{p})}{C^{(2)}(t; \vec{p}) C^{(2)}(\tau; \vec{p}) C^{(2)}(\tau - t; \vec{p})}. \quad (5)$$

For the nucleon axial charge g_A , the asymptotic plateau value as $\tau \rightarrow \infty$ yields the bare matrix element. Renormalization factors Z_A were determined non-perturbatively using the RI'-MOM scheme on the lattice and matched to $\overline{\text{MS}}$ at $\mu = 2.0$ GeV.

Extrapolating to the continuum limit via a joint fit of the form:

$$f(a, m_\pi) = f_0 + c_a \left(\frac{a}{a_0}\right)^2 + c_\pi \left(\frac{m_\pi}{m_{\pi, \text{phys}}}\right)^2 + c_L \frac{e^{-m_\pi L}}{(m_\pi L)^{3/2}}, \quad (6)$$

we obtain our final physical predictions:

$$g_A = 1.268 \pm 0.009_{\text{stat}} \pm 0.011_{\text{sys}}, \quad (7)$$

$$f_\pi = 130.4 \pm 0.6_{\text{stat}} \pm 0.7_{\text{sys}} \text{ MeV}. \quad (8)$$

These results are in excellent agreement with experimental world averages while providing unprecedented precision on non-perturbative parameters required for Meridian Collider phenomenology.

5 Summary and Outlook

We have presented a non-perturbative calculation of hadron structure observables using high-statistics lattice QCD simulations on $N_f = 2 + 1 + 1$ HISQ ensembles. Through meticulous control of excited states

and continuum extrapolations, sub-percent precision has been attained for key decay constants and form factors.

Future work will expand these calculations to flavor-singlet parton distributions and transverse-momentum-dependent (TMD) structure functions utilizing next-generation exascale compute allocations at the Apex Supercomputing Center.

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References

- [1] E. Vance, M. Sterling, and J. Thorne, *Non-perturbative lattice QCD at physical mass*, Phys. Rev. D **109**, 054501 (2024).
- [2] M. Sterling et al. (Nexa Collaboration), *Hadron form factors from HISQ configurations*, PoS **LATTICE2023**, 142 (2023).
- [3] D. K. Chen and C. Rey, *High-precision renormalization in RI-MOM scheme*, J. High Energy Phys. **04**, 088 (2025).
- [4] J. Thorne, *Algorithmic developments for GPU lattice solvers*, Comput. Phys. Commun. **275**, 108310 (2022).
- [5] Meridian Collider Collaboration, *Physics opportunities at the Meridian High-Energy Facility*, Report No. MCC-2025-01 (2025).
- [6] K. Symanzik, *Continuum limit and improved action in lattice field theory*, Nucl. Phys. B **226**, 187 (1983).
- [7] E. Follana et al. (HPQCD Collaboration), *Highly improved staggered quarks on the lattice*, Phys. Rev. D **75**, 054502 (2007).