Math 201: Linear Algebra Homework

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Exercise 1. Prove that the eigenvalues of a symmetric matrix are real.

Proof. Let A be a symmetric real matrix, so $A = A^T$. Let λ be an eigenvalue of A with corresponding non-zero eigenvector v. Then $Av = \lambda v$. Taking the complex conjugate and transpose:

$$v^* A^T = \bar{\lambda} v^*$$

Since $A = A^T$ and A is real:

$$v^* A v = \bar{\lambda} v^* v$$

Also:

$$v^*(Av) = v^*(\lambda v) = \lambda v^* v$$

Therefore, $\lambda v^* v = \bar{\lambda} v^* v$. Since $v \neq 0$, $v^* v \neq 0$, we have $\lambda = \bar{\lambda}$, meaning λ is real. □